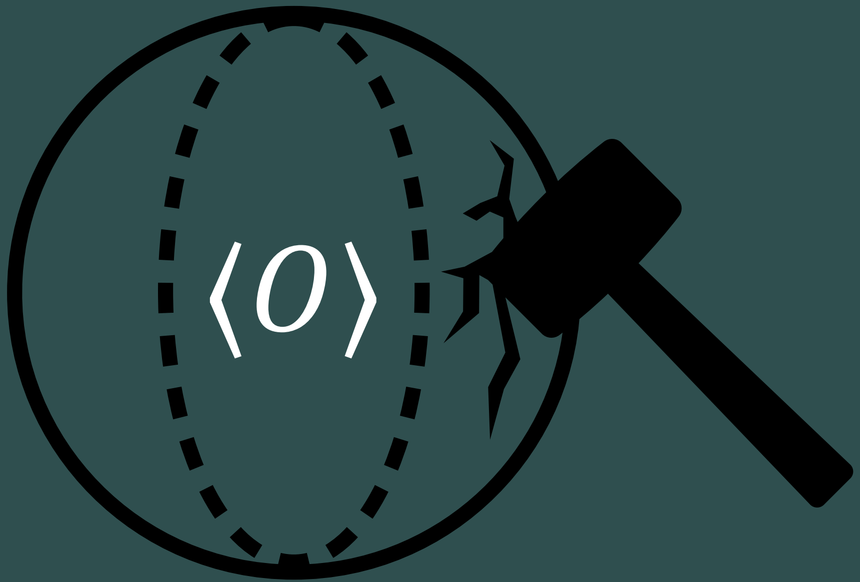
Quantum Noise Deconvolution

PhD EOY Seminars @ University of Pavia

Program in Physics 37° Cycle



30 September 2022

UNIVERSITÀ DI PAVIA

Candidate Simone Roncallo

Supervisors Chiara Macchiavello and Lorenzo Maccone

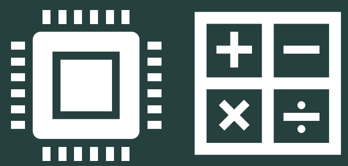
Quantum Information Theory Group (QUIT)

Quantum Information?

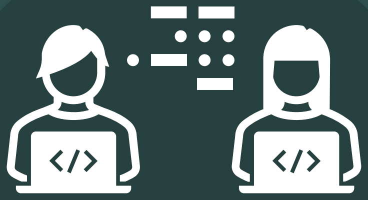
"… the study of the information processing tasks that can be accomplished using quantum mechanical systems"

>> Nielsen and Chuang *"Quantum Computation and Quantum Information"* (Cambridge 2010)

Why?



Computation



Communication
& Cryptography

Metrology

AI

and others…

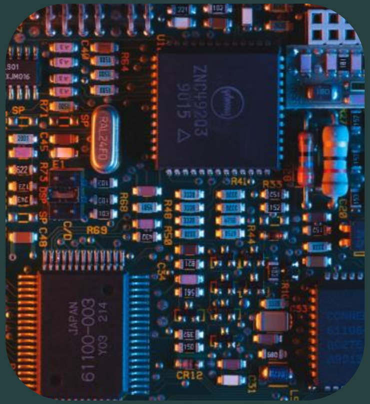
Information Bricks

Nielsen and Chuang *"Quantum Computation and Quantum Information"* (Cambridge 2010)

Bit $\in \{0, 1\}$ → Classical systems encoding

Quantum effects are suppressed!

Current



Magnetization

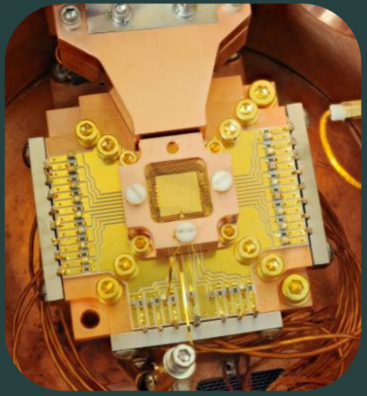
Modulation

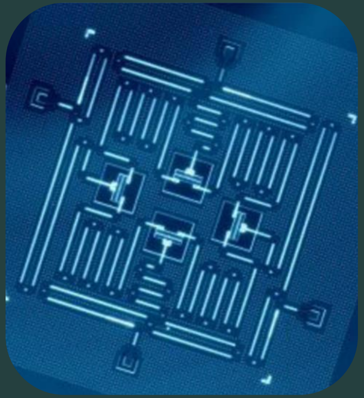
Credits: Unsplash (Umberto, Frank R, Denny Müller)

Qubit $\in \mathbb{C}^2$ → Quantum systems encoding

Two-level quantum systems!

Hyperfine levels





Cooper pairs

Polarization

Credits: NIST, IBM, Xanadu

Quantum Algorithms

Nielsen and Chuang *"Quantum Computation and Quantum Information"* (Cambridge 2010)

Preparation^n-qubits^

$$\rho = \sum_{i,j=0}^{d-1} \rho_{ij} \, |i\rangle\langle j|$$

Classical or Quantum source

$|i\rangle \in \mathcal{H}^n$

$d = 2^n$

Gates^Unitary operations^

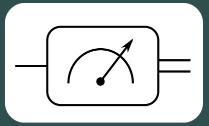
$$\rho \to U \rho U^\dagger$$

$$U U^\dagger = \mathbb{1}$$

e.g., Pauli, phase, Hadamard, CNOT…

Measurement^z-basis^

The system collapses in a **Z-basis eigenstate**



$\to |i\rangle\langle i|$

Probabilistic outcome

Ingredients

$$|0\rangle \leftrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

Superposition & Interference

$$|00\rangle \to \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

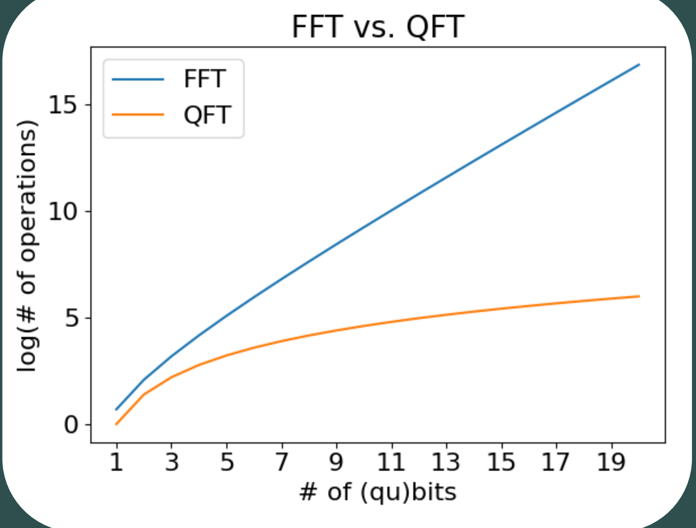
Entanglement

Example

FFT vs QFT

$O(n2^n)$ vs $O(n^2)$

Quantum advantage!

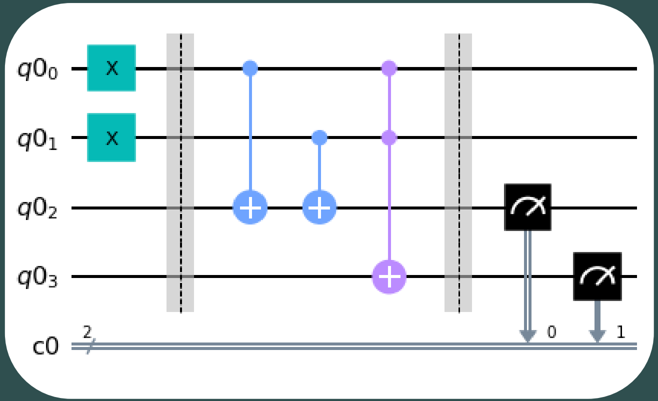


1+1 = 2? Noise

Nielsen and Chuang *"Quantum Computation and Quantum Information"* (Cambridge 2010)

Try with 01 + 01 = 10!
No advantage here, still didactically useful!

→ Quantum half adder →



NISQ era

(Noisy intermediate-scale quantum era)

Random

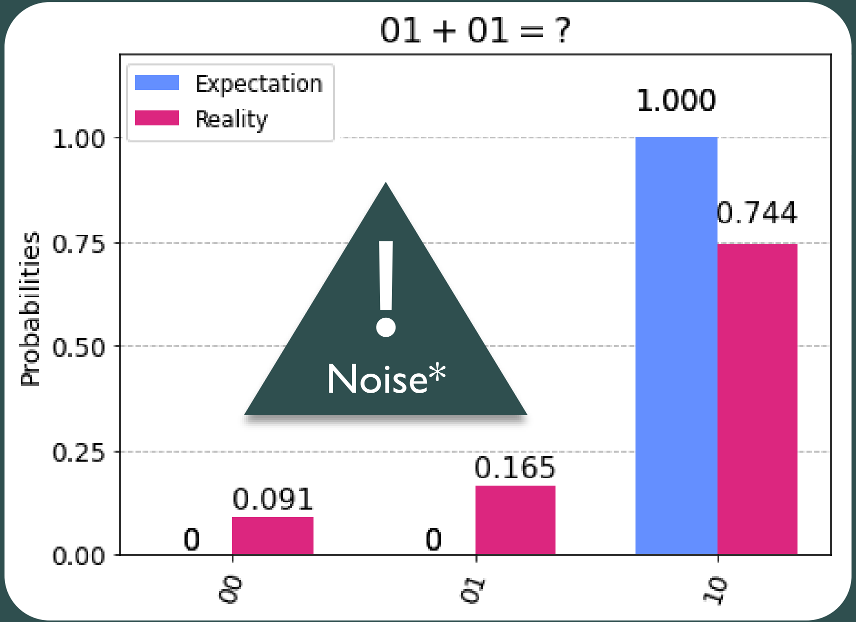
Non-unitary | Depth-scaling

Bit-flip*
$|0\rangle\langle 0| \rightarrow |1\rangle\langle 1|$
$|1\rangle\langle 1| \rightarrow |0\rangle\langle 0|$

Depolarizing
$\rho \rightarrow \frac{\mathbb{1}}{2}$

Amplitude damping
$|1\rangle\langle 1| \rightarrow |0\rangle\langle 0|$

...



Quantum Channels

Nielsen and Chuang *"Quantum Computation and Quantum Information"* (Cambridge 2010)
C. J. Wood, J. D. Biamonte and D. G. Cory *Quant. Inf. Comp. 15, 0579-0811* (2015)
C. Macchiavello and M. F. Sacchi *Phys. Rev. A 94, 052333* (2016)

Noise is described by **quantum channels**

Definition

Linear completely-positive trace-preserving (CPTP) map

$\rho \rightarrow \mathcal{N}(\rho)$

Kraus representation

$$\mathcal{N}(\rho) = \sum_i p_i A_i \rho A_i^\dagger$$

Other representations

- System-Environment
- Choi-Jamiołkowski
- Process Matrix
- Pauli-Liouville

Observables become noisy!

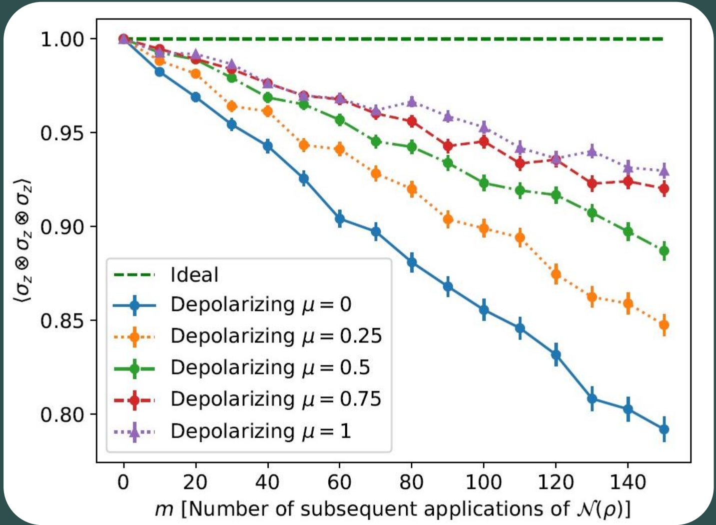
Ideal

$\langle O \rangle_\rho = \mathrm{Tr}(O\rho)$

Output

$\langle O \rangle_{\mathcal{N}(\rho)} = \mathrm{Tr}[O\mathcal{N}(\rho)]$

Effect of correlated depolarizing noise on three qubits



Error Correction

S. J. Devitt, W. J. Munro and K. Nemoto *Rep. Prog. Phys. 76 076001* (2013)

Correction

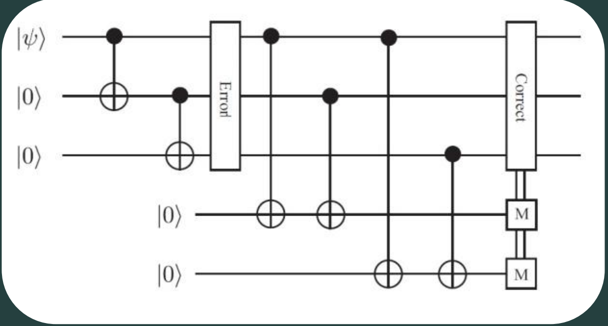
Redundancy generation + Syndrome extraction

Add redundancy — Check for errors

- ✓ Active operation (on-the-fly)
- ✓ One-shot correction
- ✓ Final state protected from errors

Result
$\mathcal{N}(\rho) \to \rho$

- ✗ Dimension increases



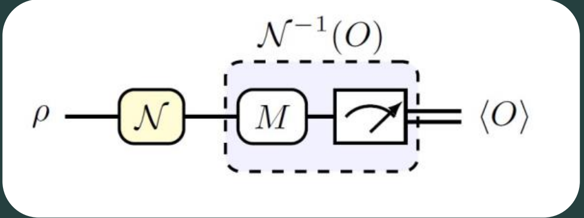
Noise Deconvolution

S. Mangini, L. Maccone and C. Macchiavello *arXiv:2112.03043v2 [quant-ph]* (2021)

Noise deconvolution
$\langle O \rangle_\rho = \langle \mathcal{N}^{*-1}(O) \rangle_{\mathcal{N}(\rho)}$

- ✓ Noiseless result from noisy experiment
- ✓ Deconvolution $\forall n \; \forall \mathcal{N} \; \forall O$ (even in presence of correlations)
- ✓ Ancilla-free

Result
$\langle O \rangle_{\mathcal{N}(\rho)} \to \langle O \rangle_\rho$



- ✗ Multi-shot correction
- ✗ Noise-inverted operator

Noise Deconvolution

S. Roncallo, L. Maccone and C. Macchiavello *arXiv:2207.12386v2 [quant-ph]* (2022)

Channel representations

- Pauli-Liouville
- Process Matrix
- Choi-Jamiołkowski
- System-Environment
- Kraus (Operator-Sum)

→

PTM

Pauli transfer matrix

$$\Gamma_{ij}(\mathcal{N}) = \mathrm{Tr}[\,\mathcal{P}_i \mathcal{N}(\mathcal{P}_j)]$$

with Pauli basis $\{\mathcal{P}_i\}$

→

Multi-qubit noise deconvolution

$$\langle O \rangle_\rho = \sum_{i,j=0}^{d^2-1} (\Gamma^{*-1})_{ij} O_j \langle \mathcal{P}_i \rangle_{\mathcal{N}(\rho)}$$

Noise characterization

$$\rho_j = \frac{\mathbb{1} + \mathcal{P}_j}{d} \Rightarrow \Gamma_{ij} \leftarrow \langle \mathcal{P}_i \rangle_{\mathcal{N}(\rho_j)}$$

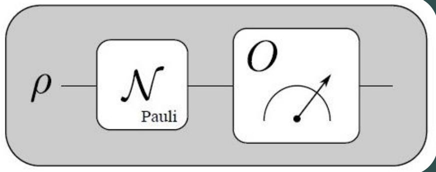
- ✓ Simplified noise inversion
- ✓ Tomographic reconstruction (post-processing)
- ✓ Complexity scales with O and $\mathcal{N}$ (up to d^2 experiments)
- ✓ Experimental estimation of the noise parameters

Noise Deconvolution

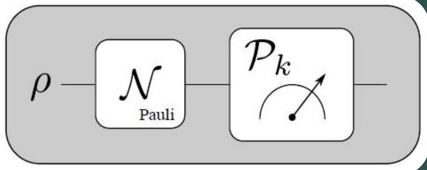
S. Roncallo, L. Maccone and C. Macchiavello *arXiv:2207.12386v2 [quant-ph]* (2022)

Deconvolution of Pauli noise

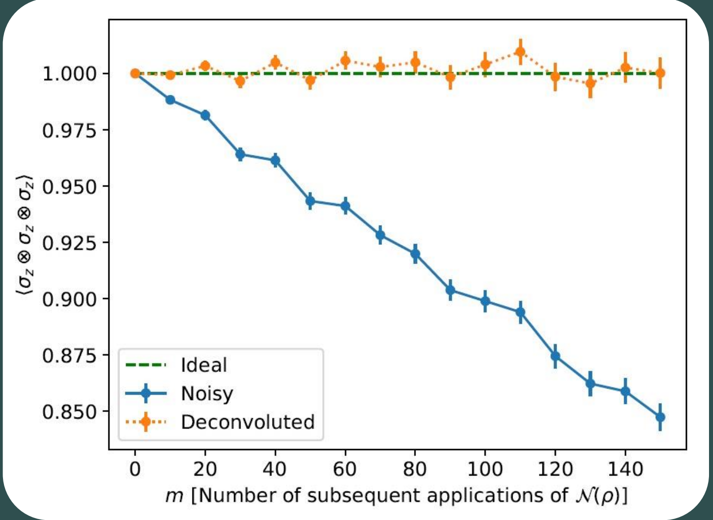
$$\langle O \rangle_{\mathcal{N}(\rho)} =$$



$$\langle O \rangle_{\rho} = \sum_{k=0}^{d^2-1} \frac{O_k}{\Gamma_{kk}} \times$$



Noisy experiment → Rescaling of the noisy data



Deconvolution of three-qubit correlated depolarizing noise

Perspectives

- Qudit extension
- Comparison with QPT
  Quantum process tomography
- Experimental realization
- Adaptive deconvolution

SQMS INFN

QUANTERA
ERA-NET Cofund in Quantum Technologies

Thank you

for your time and attention!

$$\frac{|☺\rangle + |☹\rangle}{\sqrt{2}} = |{:}){:}\rangle$$

Schrödinger's smiley

Simone Roncallo

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